CIENCIA EN REVOLUCIÓN

Isea y Lonngren, Vol. 6, Nº 17, 22–26, enero–abril, 2020.

Article:

A mathematical model that could explain the social dynamics in a country

Raúl Isea[1]*, Karl–E. Lonngren[2]
[1]Fundación Instituto de Estudios Avanzados, Hoyo de la Puerta, Baruta, Venezuela,
[2]Department of Electrical and Computer Engineering, University of Iowa, Iowa City, USA.

Recibido: enero, 2020,
Aceptado: marzo, 2020.
Autor para correspondencia: R. Isea e–mail: raul.isea@gmail.com
DOI: https://doi.org/10.5281/zenodo.3930521

Resumen

The present work presents a mathematical model that allows one to describe the social inequality of a population over time. In order to do that, the population of a nation or a region is divided into two different social groups that are labeled as Poor and Non-Poor where the social groups do not interact with each other. We later tested a second model where the Non-Poor people help the Poor people without any economic retribution. Both models appear to be stable over time. We tested both models with the data provided by the National Statistics Institute of Venezuela from 2005 to 2013.
Keywords: social inequality; system of differential equations; social classes; purchasing power; socioeconomic; Venezuela.

Artículo:

Un modelo matemático que puede explicar la dinámica social de un país

Resumen

El presente trabajo presenta un modelo matemático que permite describir la desigualdad social de una población a lo largo del tiempo. Para realizar ello, la población de una nación o región se debe dividir en dos grupos sociales diferentes que serán etiquetadas como “Pobres” y “No Pobres”. Un primer modelo es aquel donde los grupos sociales no interactúan entre sí. Más tarde probamos otro modelo donde las personas No Pobres ayudan a las personas Pobres sin ningún tipo de retribución económica. Ambos modelos son estables en el tiempo. Finalmente se corroboró dicho modelo con los datos obtenidos del Instituto Nacional de Estadística de Venezuela desde 2005 a 2013.
Palabras clave: desigualdad social; sistema de ecuaciones diferenciales; clases sociales; poder adquisitivo; socioeconómico; Venezuela.

Isea y Lonngren, Vol. 6, N° 17, 22–26, enero–abril, 2020.

1 Introduction

The concept of a social class has changed over time and it is typically based on people's purchasing power. However, it is only an approximation because there are other non-economic factors that modify the financial gap between social classes. These include the educational, artistic, and political prestige among others. Only the economic factor will be considered in this paper.

In the scientific literature there is no model that manages to explain the gap between the two social classes where one class helps the other class unidirectionally. There are models that describe the modeling of the impact of vaccination campaigns in different social groups [1], analogies between science and social classes [2], the social division based on agent models [3], the inclusion of the contributions made by the European Union on this issue [4], etc.

In this work, we describe two scenarios that will model the social gaps between Poor and Non-Poor people. In the model, we employed data from the economic indicator called "the poverty index" (in Spanish it is "Indice de Pobreza") as calculated by the National Institute of Statistics (INE) of Venezuela between 2005 to 2013 (data available at INE web page, www.ine.gob.ve).With this data, we tested two different scenarios and have been able to determine which of them is stable in time.

2 Mathematical model

We suggest that a certain system of differential equations is capable of describing the social dynamics of a population based on their economic level. The population is divided into Non-Poor and Poor people. These dependent variables are C_1 and C_2.

We assume that all people interact with each other (we will abbreviate this variable with the letter S).The social differences are characterized with six different constants. The first four constants ($\alpha_1, \alpha_2, \beta_1$, and β_2) allow one to characterize the two social groups that have or not have economic resources. The constant β_3 is the proportion of economic aid transferred from the Non–Poor to the Poor segments of the population.

The mathematical model proposed in this work is given in Equations system (1)

$$\begin{cases} \dfrac{dS}{dt} &= \Lambda - \alpha_1 S C_1 - \alpha_2 S C_2 \\ \dfrac{dC_1}{dt} &= \alpha_1 S C_1 - (\beta_1 + \beta_3 + \mu) C_1 \\ \dfrac{dC_2}{dt} &= \alpha_2 S C_2 + \beta_3 C_1 - (\beta_2 + \mu) C_2, \end{cases} \tag{1}$$

where Λ is a parameter that symbolizes the number of people entering the population regardless of whether or not they have economic resources. The constants α_1 and α_2 characterize the social groups responsible for generating the social inequality in each group, β_1 and β_2 represent their feedback with the society. The constant β_3 represents the degree of help from the population that has purchasing power to those with limited economic resources. The constant μ is *per capita* death rate.

3 Results

The methodology for solving this system of differential equations has been previously published in multiple scientific papers [5]–[9] and only the results obtained will be indicated.

First model ($\beta_3 = 0$)

In this case, there is no direct interaction between the social groups and the constant $(\beta_3) = 0$. In this particular case, two different Critical Points are obtained that are denoted as PE_{11} and PE_{12} where the first subscript indicates the scenario being considered and the second subscript lists each of the solutions that is obtained:

$$PE_{11}: \quad \left[S^\star = \frac{\beta_1 + \mu}{\alpha_1}, C_1^\star = \frac{\Lambda}{\beta_1 + \mu}, C_2^\star = 0\right],$$

$$PE_{12}: \quad \left[S^\star = \frac{\beta_2 + \mu}{\alpha_2}, C_1^\star = 0, C_2^\star = \frac{\Lambda}{\beta_2 + \mu}\right].$$

The next step is to determine if the two values are stable over time. In order to answer this, the

Isea y Lonngren, Vol. 6, N° 17, 22–26, enero–abril, 2020.

Jacobian matrix (J) of the System of Equations (1) must be calculated. We write:

$$J = \begin{pmatrix} -\alpha_2 C_2 - \alpha_1 C_1 & -S\alpha_1 & -S\alpha_2 \\ \alpha_1 C_1 & -S\alpha_1 - \mu - \beta_1 & 0 \\ \alpha_2 C_2 & 0 & -S\alpha_2 - \mu - \beta_2 \end{pmatrix}.$$

The next step is that the Jacobian should be evaluated at each of the critical points PE. The resulting Jacobian after evaluating it at PE_{11} is

$$\begin{pmatrix} -\frac{\alpha_1\Lambda}{\mu+\beta_1} & -\mu-\beta_1 & \frac{\alpha_2(\mu+\beta_1)}{\alpha_1} \\ \frac{\alpha_1\Lambda}{\mu+\beta_1} & 0 & 0 \\ 0 & 0 & \frac{\alpha_2(\mu+\beta_1)}{\alpha_1} - \mu - \beta_1 \end{pmatrix}.$$

Finally, the eigenvalues are calculated in order to establish the conditions such that the solutions are stable. This means that negative eigenvalues indicate that the modeled system is maintained over time. The eigenvalues for PE_{11} are

$$\frac{\alpha_1\Lambda + \sqrt{\alpha_1\Lambda\left[\alpha_1\Lambda - 4(\mu+\beta_1)^2\right]}}{2(\mu+\beta_1)},$$
$$\frac{\sqrt{\alpha_1\Lambda\left[\alpha_1\Lambda - 4(\mu+\beta_1)^2\right]} - \alpha_1\Lambda}{2(\mu+\beta_1)}$$

and

$$\frac{\alpha_2\beta_1 - \alpha_1\beta_2 + \mu(\alpha_2-\alpha_1)}{\alpha_1}.$$

The eigenvalues for PE_{12} are

$$\frac{\alpha_2\Lambda + \sqrt{\alpha_2\Lambda\left[\alpha_2\Lambda - 4(\mu+\beta_2)^2\right]}}{2(\mu+\beta_2)},$$

$$\frac{\sqrt{\alpha_2\Lambda\left[\alpha_2\Lambda - 4(\mu+\beta_2)^2\right]} - \alpha_2\Lambda}{2(\mu+\beta_2)}$$

and

$$\frac{\alpha_1\beta_2 - \alpha_2\beta_1 + \mu(\alpha_2-\alpha_1)}{\alpha_2}.$$

Second Model ($\beta_3 \neq 0$)

This scenario indicates that there is a unidirectional financial aid from the group of people with greater resources (Non-Poor) to those of low income (Poor) without any compensation in return. When solving the System of Equations (1), this implies that $\beta_3 \neq 0$. In this case, two PE's are obtained (PE_{21} and PE_{22}) for

$$PE_{21} : [S^\star, C_1^\star, C_2^\star];$$
$$S^\star = \frac{\beta_2+\mu}{\alpha_2},$$
$$C_1^\star = 0,$$
$$C_2^\star = \frac{\Lambda}{\beta_2+\mu};$$

for

$$PE_{22} : [S^\star, C_1^\star, C_2^\star];$$
$$S^\star = \frac{\beta_1+\beta_3+\mu}{\alpha_1},$$
$$C_1^\star = \frac{\Lambda\mu(\alpha_2-\alpha_1) + \Lambda\left[\alpha_2(\beta_3+\beta_1) - \alpha_1\beta_2\right]}{(\beta_1+\beta_3+\mu)\left[\alpha_2\beta_1 - \alpha_1\beta_2 + (\alpha_2-\alpha_1)\mu\right]},$$

$$C_2^\star = -\frac{\alpha_1\beta_3\Lambda}{(\beta_1+\beta_3+\mu)\left[\alpha_2\beta_1 - \alpha_1\beta_2 + (\alpha_2-\alpha_1)\mu\right]}.$$

Repeating the same procedure that was used previously, the eigenvalues for PE_{21} are:

$$-\frac{\alpha_2\Lambda + \sqrt{\alpha_2\Lambda\left[\alpha_2\Lambda - 4(\mu+\beta_2)^2\right]}}{2(\mu+\beta_2)},$$

$$\frac{\sqrt{\alpha_2\Lambda\left[\alpha_2\Lambda - 4(\mu+\beta_2)^2\right]} - \alpha_2\Lambda}{2(\mu+\beta_2)}$$

and

$$\frac{-\alpha_2(\beta_3+\beta_1) - \alpha_1\beta_2 + \mu(\alpha_2-\alpha_1)}{\alpha_2}.$$

It was only possible to determine analytically a self-value for PE_{22}

4 Example with Venezuela data

Numerical data obtained from the National Institute of Statistics (INE, www.ine.gob.ve) which is the government entity responsible for managing population data in Venezuela is analyzed using this model. In their analysis, they present an indicator called the "Poverty Situation" and they have classified the population in Poor and Non–Poor categories based on surveys in each household. The

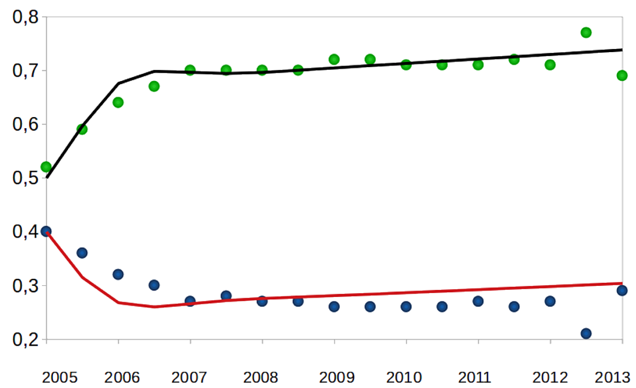

Figure 1: Semiannual data of the Non–Poor (green circle) and Poor population (blue circle) in Venezuela from 2005 to 2013. The black and red lines represent the value obtained after an adjustment of the parameters by least squares for both the Non–Poor (C_1) and Poor population (C_2) described by Equations system (1), respectively.

data is measured two times in the year (biannual). To simplify this data, we divided each population by the number of people surveyed.

The first step was to adjust each of the model parameters with this data (see Figure 1). For to do that, a Python program was written that performs a minimum square adjustment and the results are indicated in Table 1. It is important to indicate that these adjustments were made separately for each of the models described here.

Table 1: Values resulting from the adjustment by least squares of Equations system (1) for each model independently (both model 1 and model 2), with the "Poverty data" obtained from the website of the INE of Venezuela.

Model 1	Model 2
$\Lambda = 1.22$	$\Lambda = 1.22$
$\mu = 0.013$	$\mu = 0.013$
$\alpha_1 = 5.79 \pm 0.48$	$\alpha_1 = 5.71 \pm 0.72$
$\alpha_2 = -8.29 \pm 0.91$	$\alpha_2 = -8.26 \pm 0.98$
$\beta_1 = 4.09 \pm 0.32$	$\beta_1 = 3.90 \pm 0.24$
$\beta_2 = -5.81 \pm 0.64$	$\beta_2 = -5.54 \pm 0.25$
	$\beta_3 = 0.098 \pm 0.009$

5 Conclusions

In the present work, a mathematical model has been proposed that can describe the social inequality of a population as a function of time that is formed by two social groups that possibly possess economic resources. It is surprising that both models are stable over time if the population is simply divided as Poor and Non-Poor according to data obtained by the National Statistics Institute of Venezuela.

The next step will be to be able to model social behavior with more social groups such as extreme poverty, middle class, etc., in order to design possible social strategies in which the groups can interact without any economic imbalance between their members.

References

[1] J.D. Munday, A.J. van Hoek, W.J. Edmunds, and K.E. Atkins. Quantifying the impact of social groups and vaccination on inequalities in infectious diseases using a mathematical model. *BMC Medicine*, 16(1):162–(1–12), 2018.

[2] A. Bejan and M.R. Errera. Wealth inequality: The physics basis. *Journal of Applied Physics*, 121:124903–(1–8), 2017.

[3] I. Borsi and M. Primicerio. Mathematical models for social and economic dynamics and for tax evasion. *Vietnam Journal of Mathematical Applications*, 12:25–48, 2014.

[4] M. Staníčková, L. Melecky, and L. Fojtíková. *Modeling social behavior and its applications*, chapter Chapter 3: The application of social inequality models on selected regions of the european union. Nova Science Publishers, New York, 2018.

[5] R. Isea and K.E. Lonngren. On the mathematical interpretation of epidemics by kermack and mckendrick. *General Mathematics Notes (GMN)*, 19:83–87, 2013.

[6] R. Isea. Una aproximación matemática de la acción de la radioterapia en células cancerosas.

Revista Electrónica Conocimiento Libre y Licenciamiento, 11(6):150–160, 2015.

[7] R. Isea and K.E. Lonngren. A preliminary mathematical model for the dynamic transmission of dengue, chikungunya and zika. *American Journal of Modern Physics and Application*, 3:11–15, 2016.

[8] R. Isea. Análisis matemático de la difusión de un rumor entre dos grupos de personas. *Revista Electrónica Conocimiento Libre y Licenciamiento*, 8(5):85–89, 2014.

[9] R. Isea and K.E. Lonngren. A new variant of the seiz model to describe the spreading of a rumor. *International Journal of Data Science and Analysis*, 3(4):28–33, 2017.